The Conduction Analogue

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The conduction analogue

experiment: to determine using a hydraulic heat conduction analogue, the instantaneous temperature distribution in a slab material when a step change of surface temperature takes place.

INTRODUCT TO THE ANALOGY

Unsteady thermal conduction in solids of simples shape can be predicated by exact analysis, or by numerical analysis. Alternatively, the process can be represented by an analogue, usually the choice lying between an electrical or a hydraulic system.
The hydraulic analogue is attractive for teaching purpose because it gives a pictorial representation of the temperature variation with time and distance in the material.
The heat conduction in the solid is represented by liquid flow in the analogue liquid flows through small bore tubes, the flow is laminar, so that the fall in head is proportional to volume flow rate. the analogy therefore uses:

Fluid head	to represent	Temperature.
Fluid flow rate	to represent	rate of heat conduction.
Viscous resistance of small bore tubes	to represent	Thermal Resistance of the solid.

DESCRIPTION OF THE APPARATUS:

Front and rear views of the analogue are illustrated in fig 1 and fig 2. the fluid circuit is shown diagrammatically in fig 3
The analogue consists essentially of a row of vertical glass columns of equal cross sectional area. They are open at the top and at the bottom. Adjacent columns are connected by small bore tubes, each outer column is also connected at the bottom to a reservoir. E and F in fig 3. whose height can be varied and in which the liquid level is maintained constant relative to the reservoir by an overflow weir, the reservoir thus control the input and output levels. the small bore tubes connecting the two outer column to column. taps A and B control the flow rate from the fixed supply header G to reservoir E and F. the supply header is fed with fluid from the lower sump tank by a small eclectically driving pump.

Again a constant head of fluid in this tank is ensured by an overflow weir in the tank leading directly back to sump. A and B should be adjusted so that the fluid levels in E and F never fall below the overflow weirs. Taps C and D should normally be kept fully open except for either of the side reservoirs. The level of the fluid in the columns represents the temperature at various points through a parallel slab of solid material, the level in the two reservoirs representing the surface temperatures of the slab.

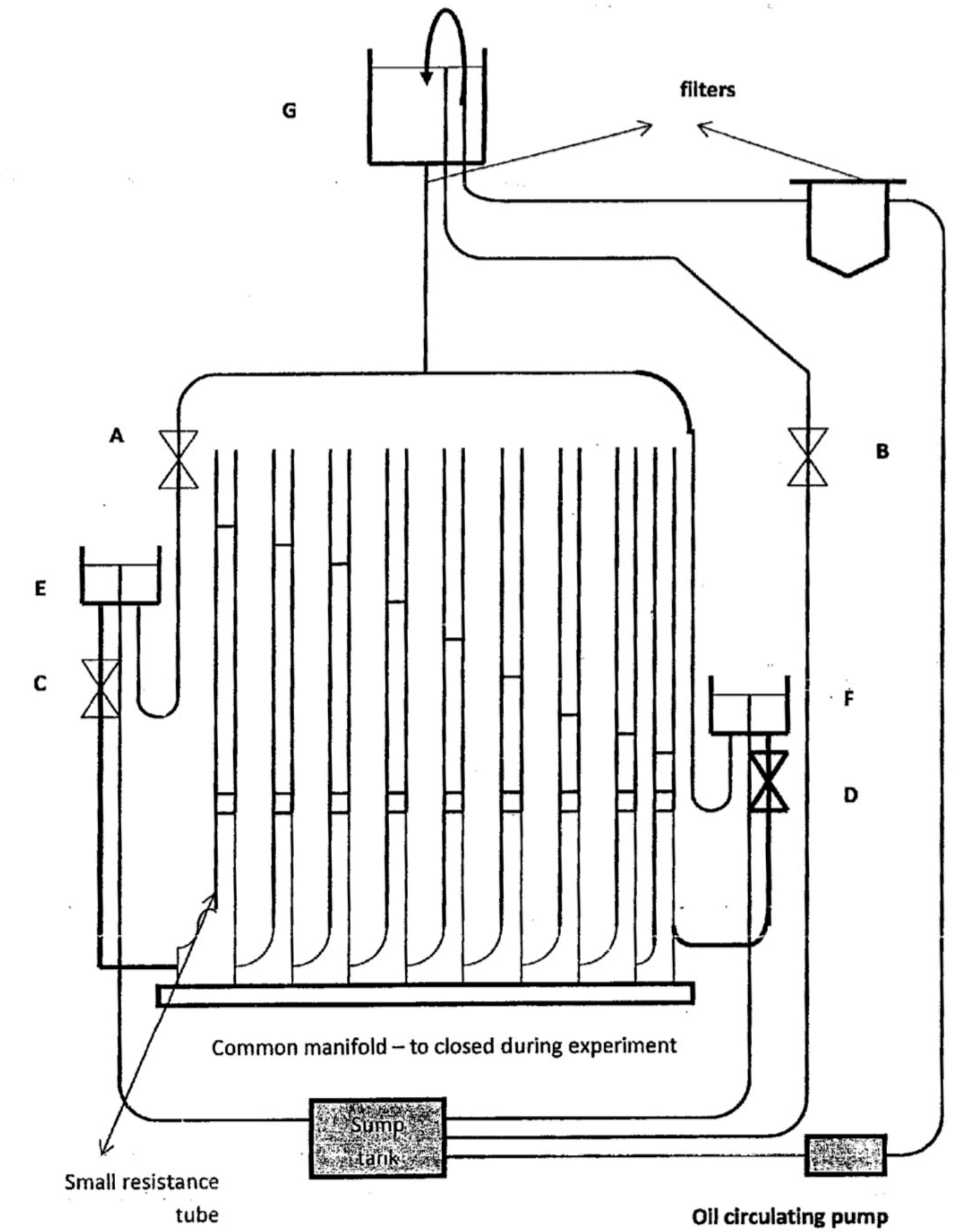
The tubes connecting the columns, and the outer columns and the header tanks E and F, are all connected via valves to a common manifold running across the apparatus below the columns. This arrangement can be used to bring the liquid in the columns to a common level quickly. This is done, if necessary, before a test is started.
The right hand tank F can be fixed at any chosen height, so that the liquid level in F represents a fixed surface temperature for a slab. Alternatively, it can be moved instantaneous up or down to simulate a step change of surface temperature.
The left hand tank E can be oscillated up and down with simple harmonic motion, so as tp represent a periodic variation of slab surface temperature. this is achieved by connecting the tank through a pantograph to a scotch yoke mechanism mounted behind the front panel and driven through a double reduction gear box by an electrical motor.

The rang of travel of the tank E can be varied between 0.31 and 0.75 meters by adjusting the throw of the eccentric behind the tube board.

The frequency of oscillation is controlled at the front of the apparatus. the approximate frequency being indicated on the meter at the left hand side of the writing table. During a test with periodic variation of level it is advisable to measure the period of oscillation more accurately during the experiment with a stop clock or watch.
The liquid levels in the tanks E and F are indicated on the front panel by pointers attached to the tanks. These pointers are coincident with the free surface fluid levels within the headers.
A space is left between the glass columns and the vertical board so that a sheet of card covered with graph paper can be inserted behind the glass columns.

During an experiment the liquid levels in the columns and the position of the pointer on tanks E and F are recorded at chosen times by a pencil mark on the graph paper ,
For consists operation the analogue fluid must be kept clean. for this reason, a filter is incorporated in the fluid circuit, and is fixed behind the vertical board carrying the columns. Before operating the apparatus, it is advisable to bleed off any air trapped in the filter by means of the bleed screw at the top of the filter unit.

Apparatus



filters
G
A
B
E
C
F
D
Common manifold – to closed during experiment
Sump tank
Small resistance tube
Oil circulating pump

THEORY OF THE ANALOGUE

In fig 4 is shown diagrammatically three columns of the analogue and three corresponding elements of the solid, which for analysis purpose is divided into laminas of equal thickness.

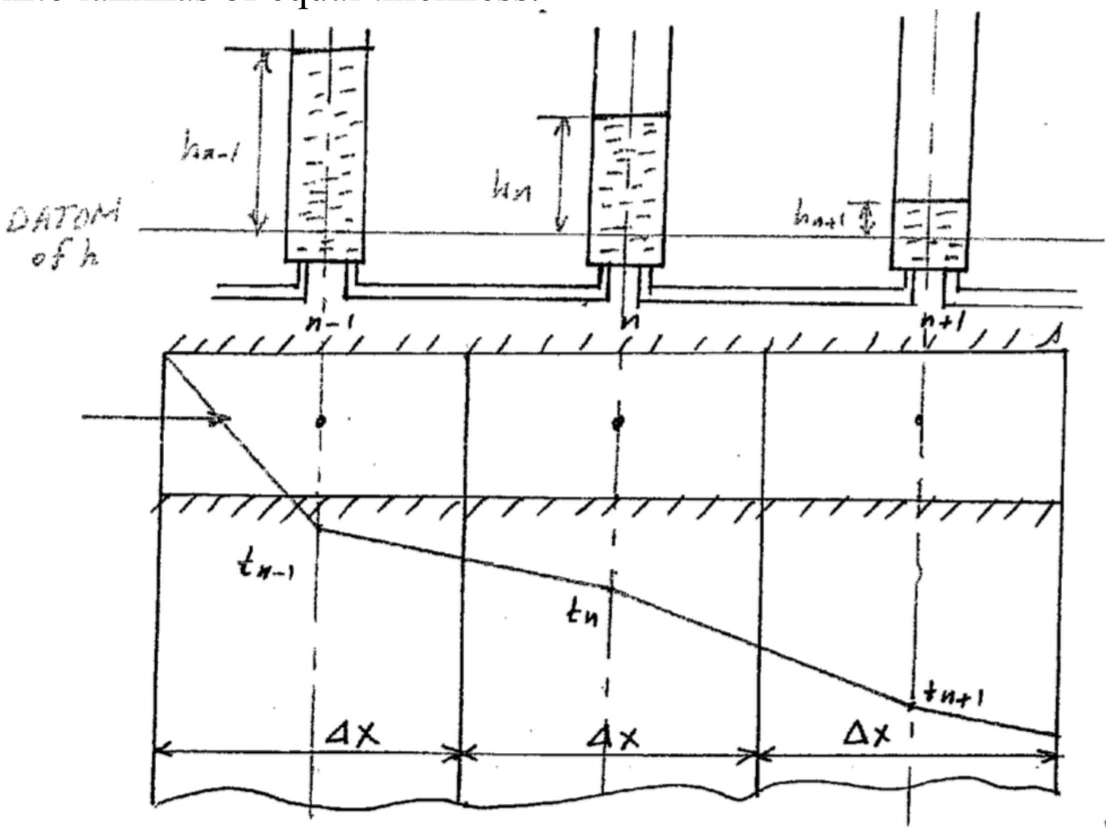


FIG.4

for the solid material	for the analogue
Let Δx = thickness of an element	Let A = x - cross section area of glass column
ρ - density	B = volume flow rate
Cp = specific heat	h = head of fluid in column
t = temperature	τ_A = time elapsed
α = thermal diffusivity	
τ_S - time elapsed	

Finite Difference Equations:

Let n-1 . n. n+1. be the number of three elements of solid under consideration in fig 4 and also the numbers of the corresponding columns of the analogue. these will be used as subscripts for t and h to indicate to witch element or column they refer.

For the solid

In the solid the material is divided into slice of equal thickness ΔX . and through them we consider heat conduction along a path of unit cross sectional area. We can write an expression. in term of tn-1 , tn and tn+1 for the rate of heat conduction into the element number n, and therefore

$$\frac{k}{\Delta x}[(t_{n-1} - t_n) - (t_{n+1} - 2t_n)] = \rho . Cp\,\Delta x\,\frac{\Delta t_n}{\Delta \tau}$$

Therefore

$$t_{n-1} - t_{n+1} - 2t_n = \frac{(\Delta x)^2}{\alpha \Delta \tau}\,\Delta t_n \;\ldots\ldots\ldots\ldots (1)$$

Where Δt_n is the increase in t_n in time increment $\Delta\tau$

$\frac{\alpha\Delta\tau}{(\Delta x)^2}$ is called the FOURIER number, symbol Fo.

The fourier number for other application can be based on any other tine period of interest, instead of $\Delta\tau$, and on any other length of interest, instead of ΔX.

For the analogue

In the analogue we consider the net rate of fluid flow into column number n. this rate is compact from the difference in head between column number n, and the two adjacent columns, and equated to the rate of accumulation of fluid in column n.

Therefore $B[(h_{n-1} - h_n) + (h_{n-1} - h_n)] = A.\frac{\Delta h_n}{\Delta \tau_A}$

Therefore $h_{n+1} - h_{n-1} - 2h_n = \frac{A}{B\,\Delta\tau_A}\,.\Delta h_n \ldots\ldots\ldots\ldots (2)$

Where Δh_n is the increase in h_n in time increment $\Delta\tau_A$.

The similarity of equations 1 and 2 forms the basis of the analogy .the variation of heads in the columns in time $\Delta\tau_A$ will be similar to the variations of temperature in the elements of the solid in time $\Delta\tau_s$, providing that.

$$\frac{(\Delta x)^2}{\alpha\Delta\tau} = \frac{A}{B\Delta\tau_A} \ldots\ldots\ldots\ldots (3)$$

this given the time scale of the analogue relative to the solid

$$\frac{\tau_A}{\tau_s} = \frac{A\alpha}{B(\Delta x)^2} \ldots\ldots\ldots\ldots (4)$$

Any convection scale factor between head and temperature can be chosen. In the analogue the change of level from some chosen datum fluid level. e.g. initial level. corresponds to change of temperature from some initial temperature.

If the analogue has n columns representing a slab of thickness L, each column will represent a lamina of the solid having thickness (L/N).
if the column are number 1, 2, 3,..etc the liquid level in column N will represent the temperature in the solid a X = (n-l/2) (L/n) from the surface of the slab and the time scale of the analogue will be:

$$\frac{\tau_A}{\tau_s} = \frac{A\alpha}{B\left(\frac{L}{N}\right)^2}$$

The analogue can therefore be used to represent a slab of any material of any thickness, without changing the characteristics. Obviously the only change when the analogue is used to represent different combinations of thickness and diffusivity is the time scale.

Value of A/B, at 20°C may be taken as 2245 and decrease this value by 2.4% for each 1°C rise in temperature. <u>For more accurate value of A/B , calibration the apparatus is necessary.</u>

Procedure

Set both header tanks E and F near the bottom of their travel and at the same height. with the circulating pump switched on bring all liquid levels in the columns to the height of the liquid in tanks E and F by using the manifold at the bottom of the columns, then close all the manifold valves.
Isolated tank F by closing tap D fig 3. move the tank £ up through the step height chosen and clamp it. Open tap C fully and instantaneously start a stop clock or watch. the liquid level in die columns should be recorded on the graph paper by pencil every half minute for the first two minute, then every minute for two minutes. The time intervals can be increase progressively as through necessary. In the first five minutes rises in only the first five columns will be detected. The significant changes of level will have taken place in an hour. And there is little point in continuing for a longer time.
All levels should first be recorder on the graph paper behind the tubes. and then the results transferred to a table. the first column of the table is used for recording the height of the moving header tank, and the other columns of the table are allocated to the written on one line. For a step change input it is usually convenient to tabulate the variation of the level from some initial steady state value.

HINTS ON USING THE APPPARAUS AND RECORDING RESULTS.

If the taped leveling valves have been used reduce the fluid in each tube to an initial common level. Then after the valves on the common manifold have been closed it may be found that the liquid levels in the columns have been raised slightly due to the valve movement. For larger step changes of input level the effect this initial error ion the liquid levels is insignificant, but if extreme accuracy is required for small step changes it may be advisable to allow the levels to reach a uniform height before starting the test.

Graphs:

1. plot a graph showing the variation of fluid height (in cm) against time.

2. By using the scale factor for each scale, plot another graph between the temperature rise against time in seconds.

Calculations

at $\tau_A = 0$

$$\alpha = \frac{k}{\rho\ Cp} = \frac{1.63}{2300\ x\ 1000} = 7.0869\ x\ 10^{-7}$$
$$\Delta x = l \backslash n = \frac{0.2}{9} = 0.0223$$
$$\tau_s = \frac{(L/N)}{\alpha}\ x\ \frac{B}{A}\ x\ \tau_A$$
$$\tau_s = \frac{0.0223}{7.0869\ x\ 10^{-7}}\ x\ \frac{1}{2245}\ x\ 0 = 0$$

at $\tau_A = 5$

$$\alpha = \frac{k}{\rho\ Cp} = \frac{1.63}{2300\ x\ 1000} = 7.0869\ x\ 10^{-7}$$
$$\Delta x = l \backslash n = \frac{0.2}{9} = 0.0223$$
$$\tau_s = \frac{(L/N)}{\alpha}\ x\ \frac{B}{A}\ x\ \tau_A$$
$$\tau_s = \frac{0.0223}{7.0869\ x\ 10^{-7}}\ x\ \frac{1}{2245}\ x\ 5 = 1.562$$

at $\tau_A = 10$

$$\alpha = \frac{k}{\rho\ Cp} = \frac{1.63}{2300\ x\ 1000} = 7.0869\ x\ 10^{-7}$$

$$\Delta x = L \backslash N = \frac{0.2}{9} = 0.0223$$

$$\tau_s = \frac{(L/N)^2}{\alpha}\ x\ \frac{B}{A}\ x\ \tau_A$$

$$\tau_s = \frac{(0.0223)^2}{7.0869\ x\ 10^{-7}}\ x\ \frac{1}{2245}\ x\ 10 = 3.1256$$

at $\tau_A = 15$

$$\alpha = \frac{k}{\rho\ Cp} = \frac{1.63}{2300\ x\ 1000} = 7.0869\ x\ 10^{-7}$$

$$\Delta x = L \backslash N = \frac{0.2}{9} = 0.0223$$

$$\tau_s = \frac{(L/N)^2}{\alpha}\ x\ \frac{B}{A}\ x\ \tau_A$$

$$\tau_s = \frac{(0.0223)^2}{7.0869\ x\ 10^{-7}}\ x\ \frac{1}{2245}\ x\ 15 = 4.6884$$

at $\tau_A = 20$

$$\alpha = \frac{k}{\rho\ Cp} = \frac{1.63}{2300\ x\ 1000} = 7.0869\ x\ 10^{-7}$$

$$\Delta x = L \backslash N = \frac{0.2}{9} = 0.0223$$

$$\tau_s = \frac{(L/N)^2}{\alpha}\ x\ \frac{B}{A}\ x\ \tau_A$$

$$\tau_s = \frac{(0.0223)^2}{7.0869\ x\ 10^{-7}}\ x\ \frac{1}{2245}\ x\ 20 = 6.251$$

at $\tau_A = 25$

$$\alpha = \frac{k}{\rho\ Cp} = \frac{1.63}{2300\ x\ 1000} = 7.0869\ x\ 10^{-7}$$

$$\Delta x = L \backslash N = \frac{0.2}{9} = 0.0223$$

$$\tau_s = \frac{(L/N)^2}{\alpha}\ x\ \frac{B}{A}\ x\ \tau_A$$

$$\tau_s = \frac{(0.0223)^2}{7.0869\ x\ 10^{-7}}\ x\ \frac{1}{2245}\ x\ 25 = 7.8140$$

at $\tau_A = 30$

$$\alpha = \frac{k}{\rho\ Cp} = \frac{1.63}{2300\ x\ 1000} = 7.0869\ x\ 10^{-7}$$

$$\Delta x = L \backslash N = \frac{0.2}{9} = 0.0223$$

$$\tau_s = \frac{(L/N)^2}{\alpha}\ x\ \frac{B}{A}\ x\ \tau_A$$

$$\tau_s = \frac{(0.0223)^2}{7.0869\ x\ 10^{-7}}\ x\ \frac{1}{2245}\ x\ 30 = 9.377$$

at $\tau_A = 40$

$$\alpha = \frac{k}{\rho\ Cp} = \frac{1.63}{2300\ x\ 1000} = 7.0869\ x\ 10^{-7}$$

$$\Delta x = L \backslash N = \frac{0.2}{9} = 0.0223$$

$$\tau_s = \frac{(L/N)^2}{\alpha}\ x\ \frac{B}{A}\ x\ \tau_A$$

$$\tau_s = \frac{(0.0223)^2}{7.0869\ x\ 10^{-7}}\ x\ \frac{1}{2245}\ x\ 40 = 12.50$$

at $\tau_A = 50$

$\alpha = \frac{k}{\rho\ Cp} = \frac{1.63}{2300\ x\ 1000} = 7.0869\ x\ 10^{-7}$

$\Delta x = L \backslash N = \frac{0.2}{9} = 0.0223$

$\tau_s = \frac{(L/N)^2}{\alpha}\ x\ \frac{B}{A}\ x\ \tau_A$

$\tau_s = \frac{(0.0223)^2}{7.0869\ x\ 10^{-7}}\ x\ \frac{1}{2245}\ x\ 50 = 15.628$

at $\tau_A = 60$

$\alpha = \frac{k}{\rho\ Cp} = \frac{1.63}{2300\ x\ 1000} = 7.0869\ x\ 10^{-7}$

$\Delta x = L \backslash N = \frac{0.2}{9} = 0.0223$

$\tau_s = \frac{(L/N)^2}{\alpha}\ x\ \frac{B}{A}\ x\ \tau_A$

$\tau_s = \frac{(0.0223)^2}{7.0869\ x\ 10^{-7}}\ x\ \frac{1}{2245}\ x\ 60 = 18.75$

at $\tau_A = 70$

$\alpha = \frac{k}{\rho\ Cp} = \frac{1.63}{2300\ x\ 1000} = 7.0869\ x\ 10^{-7}$

$\Delta x = L \backslash N = \frac{0.2}{9} = 0.0223$

$\tau_s = \frac{(L/N)^2}{\alpha}\ x\ \frac{B}{A}\ x\ \tau_A$

$\tau_s = \frac{(0.0223)^2}{7.0869\ x\ 10^{-7}}\ x\ \frac{1}{2245}\ x\ 70 = 21.879$

at $\tau_A = 80$

$\alpha = \frac{k}{\rho\ Cp} = \frac{1.63}{2300\ x\ 1000} = 7.0869\ x\ 10^{-7}$

$\Delta x = L \backslash N = \frac{0.2}{9} = 0.0223$

$\tau_s = \frac{(L/N)^2}{\alpha}\ x\ \frac{B}{A}\ x\ \tau_A$

$\tau_s = \frac{(0.0223)^2}{7.0869\ x\ 10^{-7}}\ x\ \frac{1}{2245}\ x\ 80 = 25$

at $\tau_A = 90$

$$\alpha = \frac{k}{\rho\ Cp} = \frac{1.63}{2300\ x\ 1000} = 7.0869\ x\ 10^{-7}$$

$$\Delta x = L \backslash N = \frac{0.2}{9} = 0.0223$$

$$\tau_s = \frac{(L/N)^2}{\alpha}\ x\ \frac{B}{A}\ x\ \tau_A$$

$$\tau_s = \frac{(0.0223)^2}{7.0869\ x\ 10^{-7}}\ x\ \frac{1}{2245}\ x\ 90 = 28.1306$$

at $\tau_A = 100$

$$\alpha = \frac{k}{\rho\ Cp} = \frac{1.63}{2300\ x\ 1000} = 7.0869\ x\ 10^{-7}$$

$$\Delta x = L \backslash N = \frac{0.2}{9} = 0.0223$$

$$\tau_s = \frac{(L/N)^2}{\alpha}\ x\ \frac{B}{A}\ x\ \tau_A$$

$$\tau_s = \frac{(0.0223)^2}{7.0869\ x\ 10^{-7}}\ x\ \frac{1}{2245}\ x\ 100 = 31.2562$$

Graph 1: The variation of fluid height (in cm) against time.